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13 STRUCTURAL WELDING AND PLASTIC ~~ANALYSIS~~ DESIGN

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NOMENCLATURE

Symbols:

b	width of flange of WF section
b_s	width of stiffener plate
d	depth of WF section
d_w	unsupported depth of web section
α	shape factor
F	load factor, or ratio of ultimate load to working load
L	length of beam
M_p	plastic moment of a WF section
P	an axial column load
S	elastic section modulus
Z	plastic section modulus
t	thickness of flange of WF section
t_s	thickness of stiffener plate
w	beam load in Kips/foot
w	column web thickness
W	total distributed load on a beam span
X	a distance
	unit tensile or compressive stress
	yield point unit stress
	angular rotation

why not get this into the Lab system & give it a 233 report number?

watch this "new" business. It is in Canadian code and is now permitted by AISC here.

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STRUCTURAL WELDING AND PLASTIC ~~ANALYSIS~~ DESIGN

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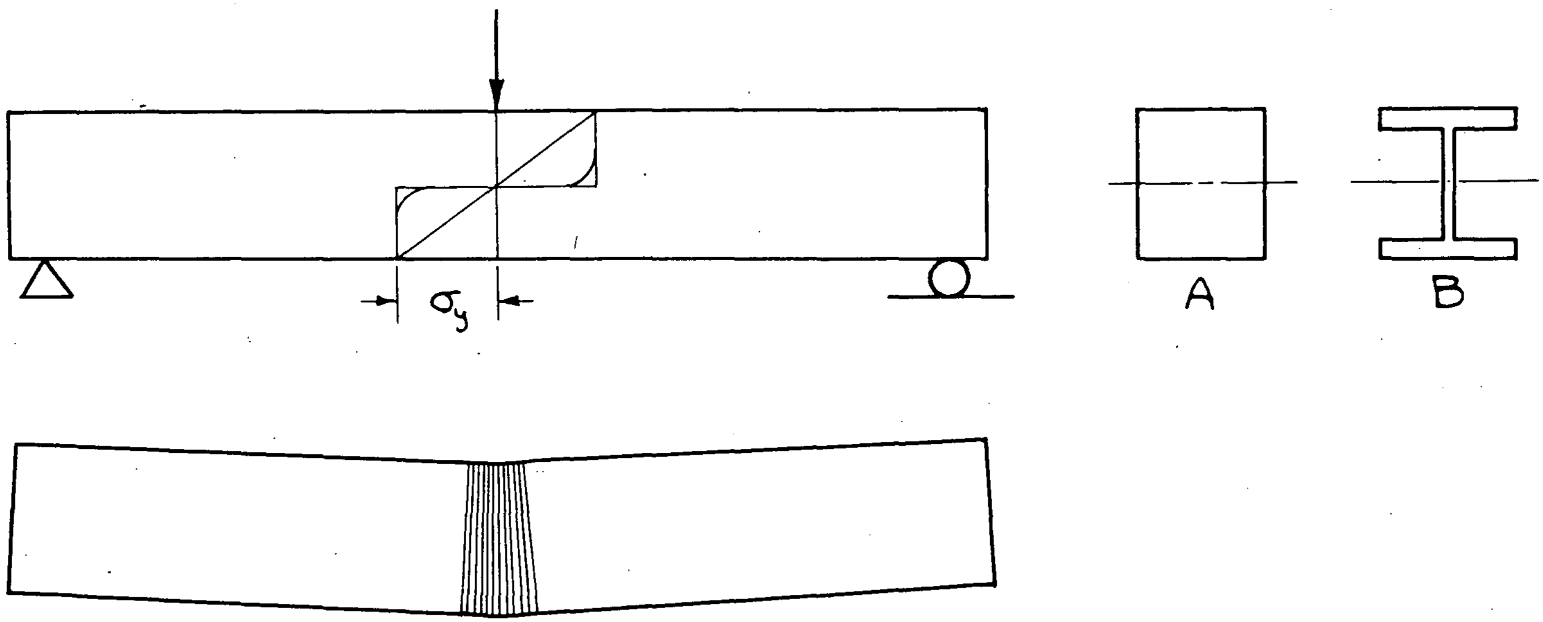
Hungary (PR-14)

An inspection of the title reveals two topics: (1) Structural Welding, and (2) Plastic Design. Since plastic design is a ~~very~~ new development in the United States and Canada, it appears necessary to explain the method in some detail to provide the necessary background for a discussion of the structural welding aspects from the viewpoint of this new concept of design.

Plastic design began (for the author and at least some of his colleagues) as an analysis method in the laboratory to predict, or to compute, the ultimate or limit load on a structure to be tested. It has been found that conventional design methods are useless in making a pre-calculation of the ultimate strength of a structure involving beams or girders unless sizable adjustments are made to the computed answers. The use of so-called "constants" to span the gap between elastic theory and the true limit loads are unsatisfactory especially when it is learned that the constants are not truly constants, just "fudge-factors". Consider the elementary case of a simple beam being tested under a central concentrated load, Fig. 1. Our computations by the elastic theory take us up to the point where the outermost fibers just reach the yield point stresses. But the beam continues to carry more load, dependent on the shape of its cross-section, until the true limit load is reached. This limit load can now be predetermined if σ_y (yield point stress

Can't make ref to other papers. See attached

See also p 1.5 of 205.53



Let S = Section Modulus (Elastic Analysis)
Let Z = Plastic Modulus = Shape Factor x S
Shape Factor is the ratio between the resisting moment of the cross-section with all fibers at σ_y and the cross-section with only the outer fibers at σ_y.

Figure 1. PLASTIC HINGE-SIMPLE BEAM

of the steel) is known. The ultimate resisting moment, M_p, for either Section A or B, Figure 1, is the first moment of the cross section about the neutral axis times σ_y. In the case of an elastic analysis the resisting moment is given by $\sigma_y S$, where S is the section modulus. The ratio of the ultimate resisting moment, and the elastic limit moment, $\sigma_y S$, is called the shape factor and will be designated as "u". It has been suggested that a new term be introduced, Z, the plastic section modulus.

This new term, the plastic section modulus, may be found by multiplying the regular section modulus by the shape factor, or, as mentioned above, it is the moment of the profile area about the neutral axis. Figure 2 gives the shape factors for several sections. For the wide flange sections the shape factor varies from about 1.11 to 1.23 with 1.14 representing a good average value.

Referring back to the simple beam in Figure 1, if one attempts to place additional load on the beam above the ultimate load, the beam will continue bending to collapse. This internal action at midspan of the beam corresponds to the development of a "mechanism", and it might be said that a "plastic hinge" has formed there. The resulting limit moment, $\sigma_y Z$, is termed the "plastic moment" of the beam, M_p.

<u>Application of Plastic Design to a Fixed Beam</u>. In Figure 3(A) there is shown the familiar loading and moment diagrams for a fixed beam uniformly loaded. To design the beam for a given load "w" lbs/ft it is first necessary to multiply the "w" by a suitable factor of safety, F. Thus the limit load is "Fw" lbs/ft. The factor of 1.85 has been advanced as providing, for gravity loading, equal safety as that for a simple beam designed by the elastic method. For a combination of wind and gravity loading the factor should be 1.40.

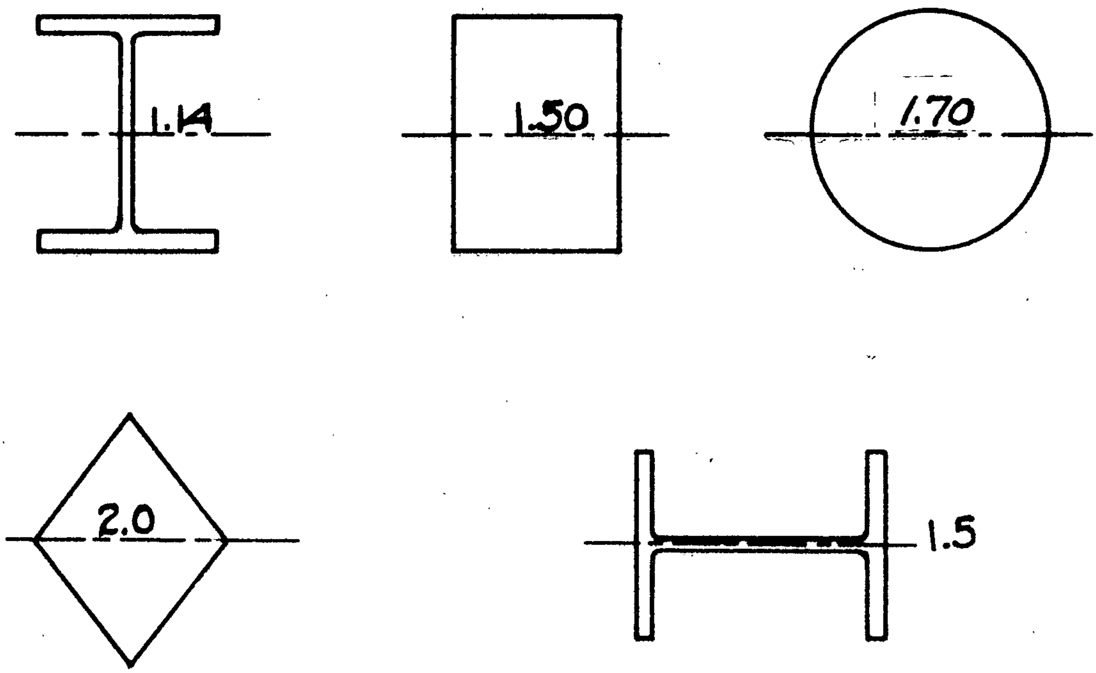


FIG. 2. SHAPE FACTORS

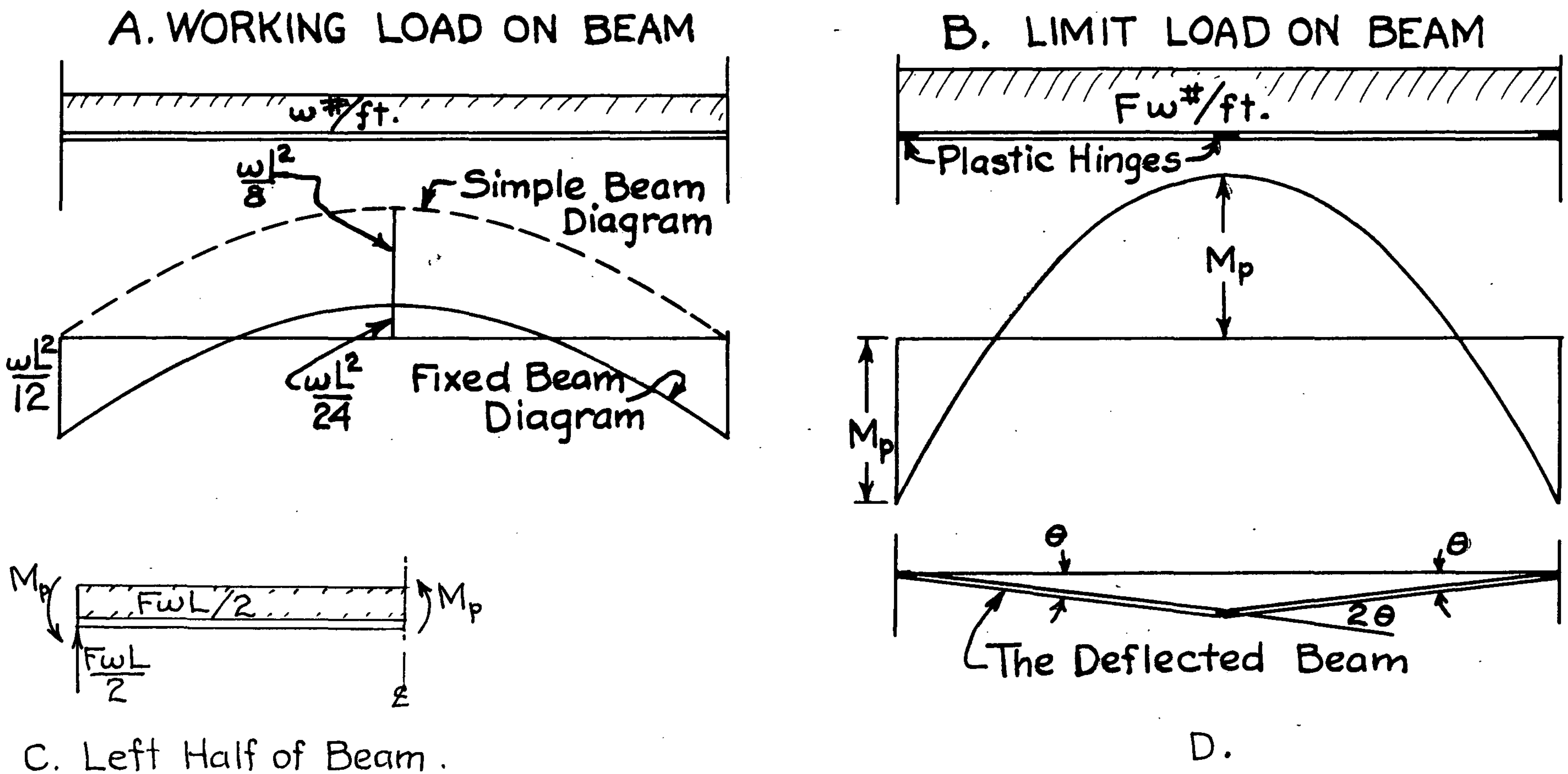


FIG. 3. APPLICATION OF PLASTIC DESIGN TO A FIXED BEAM

The next step in the design of the fixed beam is to envision the development of the plastic hinges as in Figure 3 (B). From the moment diagram it is clear that at low loads on the beam the ends of the beam are stressed to twice that at midspan. As the loads are increased the end moments eventually reach their limiting values of the plastic moments. Now as the beam load is further increased no immediate collapse of the beam occurs but rather the deflection slowly increases with the increase in load, the end moments holding steady at M_p, while the midspan moment increases until it too finally reached M_p. At that load the third plastic hinge forms (at midspan) and the ultimate load is reached. ~~collapse takes place~~. Figure 3 (B) shows the beam loaded to its limit, the formation of the three plastic hinges, the moment diagram at ultimate ~~limit~~ load, and lastly, the deflected beam (where the elastic deflections can be omitted).

There are a number of methods for computing the required plastic moment, two of which will be given. The first involves nothing but free body diagrams and the laws of equilibrium, while the second utilizes the principle of "virtual displacement".

Solution No. 1

Draw left half of beam as in Figure 3-C:

$$\Sigma M_A = 0$$

$$2 M_p = \frac{FwL}{2} \cdot \frac{L}{4}$$

whence $M_p = FwL/16$

Solution No. 2 - Virtual Displacement

"If a system of forces in equilibrium is subjected to a virtual displacement, the work done by the external forces equals the work done by internal forces". Assume in Fig. 3B that plastic hinges form at the two ends and mid-span and that the angular rotation at each end is "θ" as shown in Figure 3-D. By inspection the angular rotation at midspan is "2 θ".

Internal work = External work

$$M_p\,\theta + M_p\,2\,\theta + M_p\,\theta = FwL\ \frac{L\,\theta}{4}$$

Cancelling θ's and collecting M_p's

$$4\,M_p = FwL^2/4$$

$$\text{whence } M_p = FwL^2/16 \qquad (1)$$

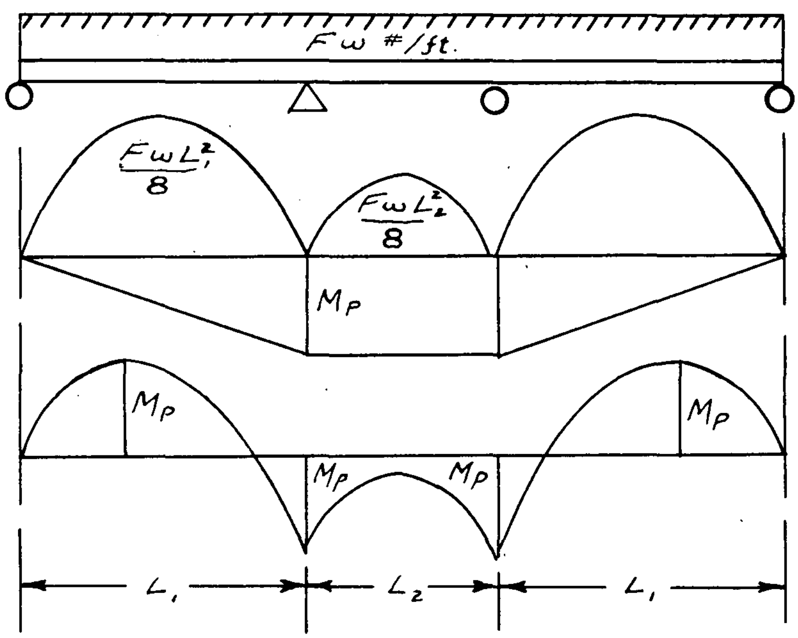
Having determined that $M_p = FwL^2/16$ the final step in the design is to select the member knowing M_p and the specification (or guaranteed) value of σ_y, which for A-7 steel is 33 ksi.

$Z = M_p/\sigma_y$ (where M_p must be in inch - Kips if σ_y = Ksi)

If w is expressed in lbs/linear ft., and L is in feet

$$Z = \frac{1.85\ wL^2}{16 \times 33} \times \frac{12}{1000} = .0427\ wL^2 \times 10^{-3}\ \text{inches}^3 \qquad (2)$$

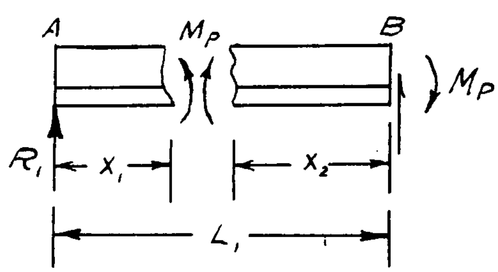
For the specific case of a uniformly distributed load of 50 Kips on a 15 ft. span the required $Z = 32.0$ in.^{3} (from which the required $S = 32.0/1.14 = \underline{28.0}$ and a <u>12WF27</u> would be safe. In a similar manner by conventional design methods:



The continuous beam of constant section with its limit load of Fw lb/ft.

Moment diagram by parts

Composite moment diagram



Free body diagrams of 1st span to locate plastic hinge.
Take moments about A & B of the respective free body diagrams.

Left free body: $M_p = Fwx_1^2/2$ (3)

Right free body: $2M_p = Fwx_2^2/2$ (4)

or $M_p = Fwx_2^2/4$ (5)

Equating (3) & (5) $\dfrac{Fwx_1^2}{2} = \dfrac{Fwx_2^2}{4}$

whence $x_1 = \dfrac{x_2}{2}$ (6)

Now $x_1 + x_2 = L_1$ (7)

from which $x_1 = L_1/2.414$ (8)

Substituting in (3) $M_p = \dfrac{FwL_1^2}{11.65}$ (9)

Figure 4 - PLASTIC DESIGN APPLIED TO A CONTINUOUS BEAM

Required simple beam: $S = 56.2$, Use 16WF36

Required fixed beam : $S = 37.5$, Use 14WF30

It may be remarked, in comparing the above three designs, that the same factor of safety exists for the simple beam and plastic designs, while the conventional design for the fixed beam has a greater safety factor.

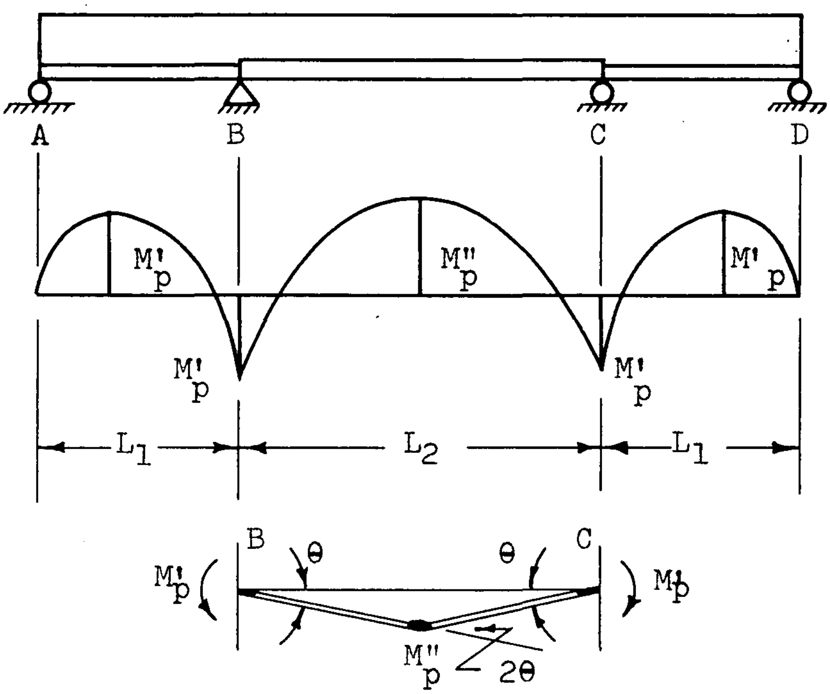
Application of Plastic Design to a Continuous Beam. The design, carried to the determination of M_p, is shown in Figure 4. By inspection of the composite moment diagram no greater moments exist elsewhere in the beam, therefore, the solution may be accepted without further checking. Should the end spans be 15 ft. and loaded with 50 Kips each, uniformly distributed, the required sectionfrom (9) is

$$S = Z/1.14 = \frac{M_p \times 12}{1.14 \times 33} = \frac{1.85 \times 50 \times 15 \times 12}{1.14 \times 33 \times 11.65} = 38.6 \text{ in.}^4$$

Use 14WF30

Continuous Beam with Long Middle Span. In this case of a long middle span and relatively short end spans the exaggerated moment diagram in Fig. 4 is reversed, the mid-span moment in the second span may exceed the maximum moment near the middle of the first span. This is shown in Fig. 5. Assume in this example that no partial length cover plates are to be employed to strengthen the beams at critical points. Under the assumption that the first span governs the design, formula (9) obtains:

$$M_p = FwL_1^2/11.65 \tag{9}$$



Assumptions:

$L_1 = L_3$

$L_1 < L_2$

$Z_1 \neq Z_2$ (Not known, however, which is the greater)

Procedure:

Due to sudden change of section at B, the lesser M_p for 1st & 2nd spans obtains there. If decision is not obvious, ascertain by use of equations (9) & (10).

Assume in example:

$$M'_p < M''_p$$

Then M'_p obtains at B & C

By virtual displacement:

$$2M'_p\,\theta + M''_p\,2\theta = FwL_2^2 \cdot \frac{L_2\theta}{4}$$

$$\text{Whence: } M''_p = \frac{FwL_2^2}{8} - M'_p \qquad (11)$$

Design 1st span using (9)

Design 2nd span using (11)

Check: M''_p in (11) must be larger than M'_p from (9)

Figure 5 - CONTINUOUS BEAM WITH DIFFERENT SECTIONS FOR END AND MIDDLE SPANS

On the other hand if the second span governs the design, the problem there is that of a beam with M_p at the ends and M_p at mid-span. In other words the problem is that of the fixed beam problem already discussed in which, from (1):

$$M_p = FwL^2_2/16 \qquad (10)$$

The greater of the two M_p's governs the design.

Continuous Beam with Different Sections for End and Middle Spans. The solution for this case is given in Figure 5. Had M_p'' been less than M_p', in the example, the solution would have been reversed in that M_p'' would have been determined first by (10). Then a derivation similar to that for (9) in Figure 4 would have to be made to determine M_p'. The free body diagram in this figure should be corrected to show M_p' at the break, and M_p'' at B. The revision is as follows:

$$M_p' = FwX_1^2/2 \qquad (12)$$

$$M_p' + M_p'' = FwX_2^2/2 \qquad (13)$$

$$\text{or } M_p' = FwX_2^2 - M_p''$$

$$\text{but } M_p'' = FwL_2^2/16 \quad \text{from} \quad (10)$$

Substituting, and equating M_p's, and cancelling Fw/2

$$X_1^2 = X_2^2 - L_2^2/8 \qquad (14)$$

$$\text{but } X_1 + X_2 = L_1$$

$$\text{whence } X_1 + \sqrt{X_1^2 + \frac{L_2^2}{8}} = L_1 \qquad (15)$$

This can be solved for X_1 by trial knowing that, if $M_p'' < M_p'$, X_1 will be slightly greater than $L_1/2.414$ as found in (8).

It is possible that the numbers are such that an approximate solution for M_p' can be safely made by use of (9), selecting a slightly oversize member, based on the knowledge that (9) is slightly on the unsafe side.

As an interesting sidelight, in considering the above problem of different sections for the end and middle spans, is that, at point B, Fig 5, an advantage acrues if the weaker section is reinforced by short plates, ordinarily only a few feet long, to bring its M_p there up to equal that in the stronger beam. From the examples given it is not felt necessary to detail further the plastic design procedure.

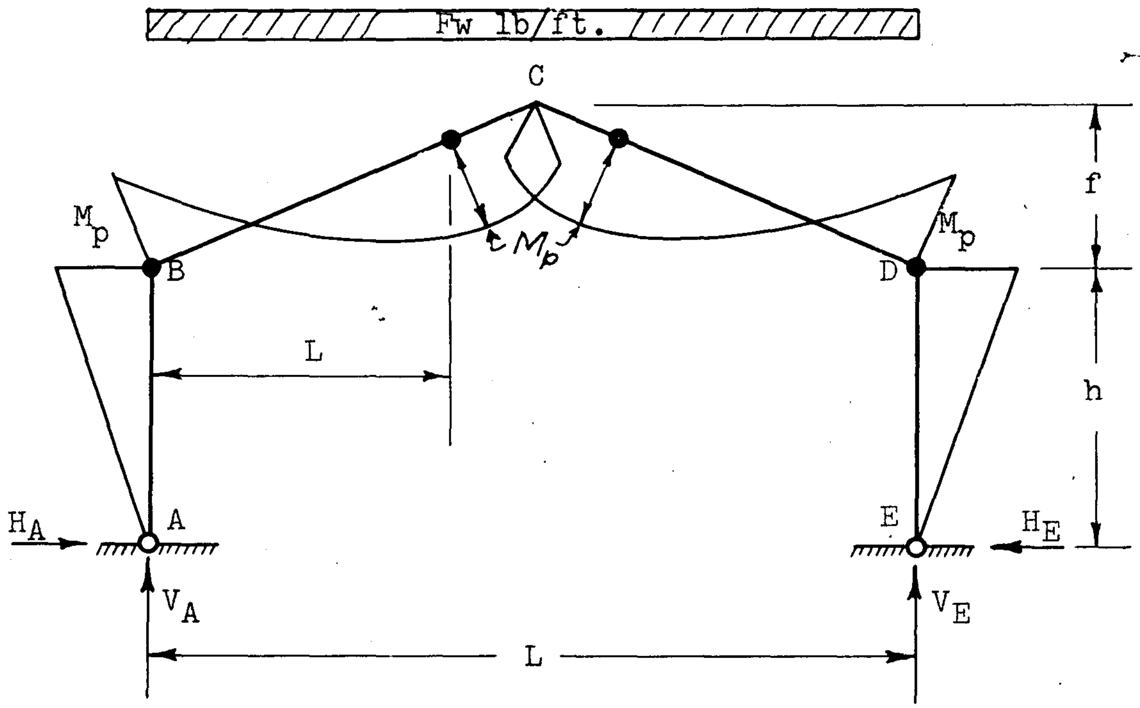
RULES. From the above examples it becomes apparent that the following conditions obtain:

1. The structure is in equilibrium with respect to the ~~limit~~ ultimate loads, and thus the rules of equilibrium for any free-body sketch may be used in design or to check the design.

2. The structure, under the ~~limit~~ ultimate load, is on the verge of becoming a mechanism.

3. The moment produced by the ~~limit~~ ultimate loading must at no point in the structure exceed the available plastic moment.

Are you sure you want to use this term? ultimate



$$V_A = V_E = FwL/2$$

$$H_A = H_E = M_p/h$$

$$Q = f/h$$

$$\alpha = \frac{1}{Q}\left[\sqrt{1 + Q} - 1\right]$$

$$M_p = \frac{FwL^2}{4}\left[\frac{\alpha(1 - \alpha)}{\sqrt{1 + Q}}\right]$$

After AISC - Ref. 20

Did you give source of these eqs.

Figure 6 - SINGLE SPAN RIGID FRAME - GRAVITY LOADING

Plastic Design Applied to Rigid Frames.

Time does not permit a complete treatment of plastic design applied to rigid frames. Briefly, if one is accustomed to elastic design methods, there are some points of similarity. The pertinent moment diagrams are similar but in the case of plastic design the key moments become M_p. Thus in the case of a single span rigid frame as in Fig. 6, from elastic analysis it is known that the probable loading to govern the design will be either the gravity loading with F of 1.85 as in Fig. 6 or gravity plus wind loading, with F reduced to 1.4 as in Fig. 7. The plastic hinges are as shown in the figures. A free body of the column reveals M_p at the top end and an axial thrust. This means that, as in elastic design, the column must be designed by an interaction formula.

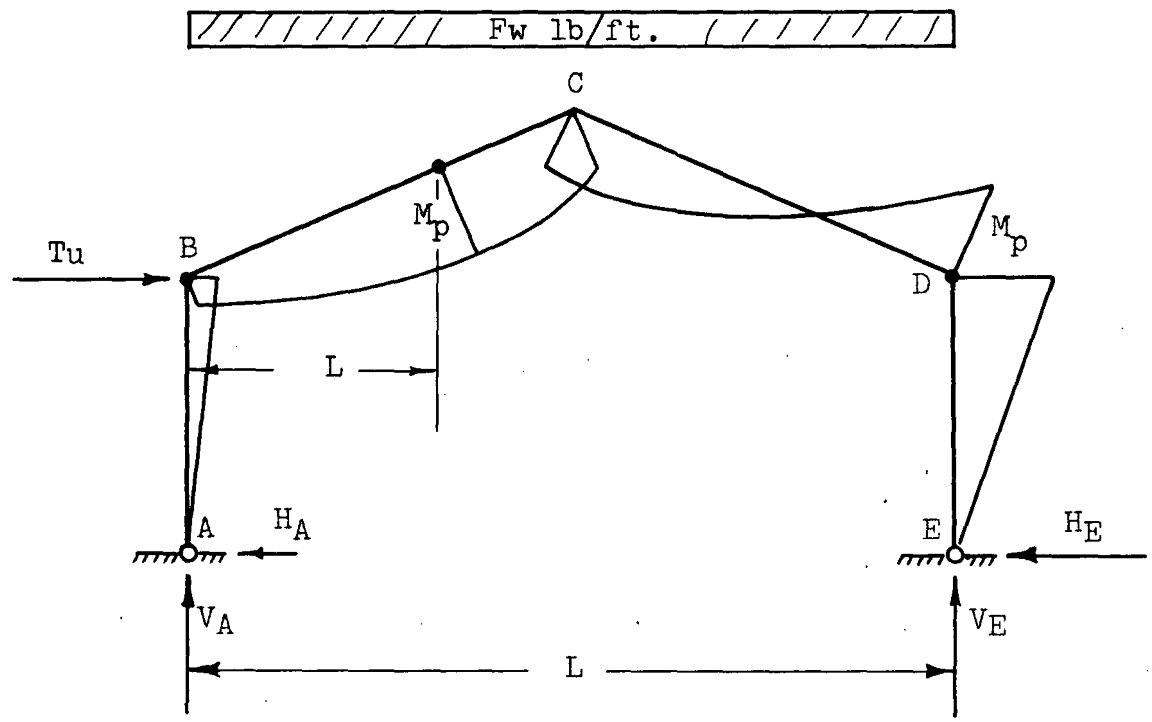
The time honored interaction formula will undoubtedly be replaced by more accurate formulas dependent on the relative sense of the moments at the ends of the members, which are subjected simultaneously to axial load and moment. For the case of a rigid frame with fixed bases, the end moments on the column have the same sense and therefore try to force the column to bend in an S - curve. ~~Research by~~ Galambos and Ketter[16] indicates the use of the formula:

$$\frac{M_o}{M_p} = 1.18 - 1.18 \left(\frac{P}{P_y}\right) \qquad (16)$$

where M_o is effective end moment reduced from M_p because of axial load

P is the axial load on the column

P_y is the product of the column area and σ_y



After AISC.- Ref. 20

$$R_A = \frac{FwL}{2} - \frac{Tuh}{L}$$

$$R_E = \frac{FwL}{2} - \frac{TuH}{L}$$

$$H_A = Tu - H_E$$

$$H_E = M_p/h$$

$$Q = f/h$$

$$C = \frac{2Tuh}{FwL^2}$$

Tuh = moment of ultimate horizontal loading taken about point A

A. (Possible Condition)

When $C > \dfrac{1}{1+Q}$

$$M_p = \frac{FwL^2 \cdot C}{4}$$

$$\alpha = 0$$

B. (Usual Condition)

When $C < \dfrac{1}{1+Q}$

$$M_p = \frac{FwL^2}{4}\left[\frac{(1-\alpha)(C+\alpha)}{(1+Q)(1-QC)}\right]$$

$$\alpha = \frac{1}{Q}\left[\sqrt{(1+Q)(1-QC)} - 1\right]$$

Figure 7 - SINGLE SPAN RIGID FRAME - GRAVITY PLUS WIND

For the case of pin-based columns, and, therefore, single curvature, and where P/P_y does not exceed 0.15, and where the L/r of the column about the strong axis is less than 60, the plastic moment need not be reduced. In more severe cases ~~a~~ formulas similar to (16) are ~~is~~ being supplied by the AISC in the form:

$$M_o/M_p = B - G \ (P/Py) \quad \text{(Case II, Pin-ended at bottom, or nearly so).} \qquad (17)$$

$$\text{and } M_o/M_p = 1.0 - K\,(\,''\,) - J(P/P_y)^2 \quad \text{(Case III, For moments of opposite sense at ends of the column)}$$

Values of B, ~~and~~ G, K, and J fluctuate with L/r and will be supplied in a tabulation. As a generalization, the values of M_o in (17) will be slightly lower than those in (16).

Fortunately for the busy designer, charts are appearing from which the pertinent loading conditions can be quickly established and the required M_p's determined with dispatch. Further, the plastic modulus, Z, has been computed for all wide-flange shapes and will soon be available.[20]

The subject of knees in rigid frames brings up some interesting questions ~~functions~~. Knees may be indicated for any of several reasons:

1. Architectural effect, - the beauty of curved knee needs no defense.
2. Economy, - weight can be saved but at the expense of design time and increased fabrication costs.
3. For field erection with high strength bolts, - ~~an enlarged section such as a triangular shaped knee provides good space for the necessary bolts.~~

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or refer to P.R. #23, 25 & Int Rep

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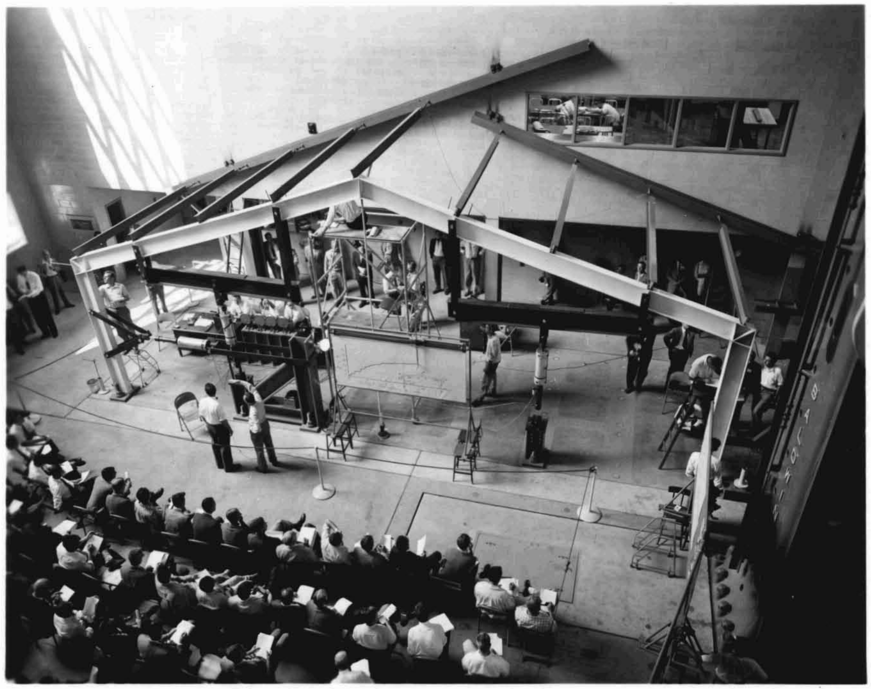
an enlarged section such as a triangular shaped knee provides good space for the necessary bolts and at the same time reduces the required size of the rafters.

An excellent design procedure for rigid frames with knees is given in the new AISC Manual, "Plastic Design in Steel"[(20)] (which at the moment is in "manuscript copy" and ~~available in~~ & should soon be generally available ~~but limited supply~~). Time does not permit further details on this subject of knees, but the thought is expressed that the bolted connection for field erection will make the knee more and more popular. The speaker has worked with one company that specializes in "packaged" factory buildings utilizing just such knees. The rigid frame sections have been neatly designed for downhand shop welding of the component parts, and for field bolting.

not available

In general, there are two arguments for the adoption of plastic design, (1) definiteness of the factor of safety, and (2) economy. The example already shown of the fixed beam gives an idea of the economy obtained. As regards definiteness of the factor of safety, many tests were made at Lehigh University this past ten or more years in which the agreement between the computed limit loads and the test results were remarkable. ~~A~~ Two ~~few~~ of the tests are shown in the photographs, Fig. 8, while Figures 9 and 10 show the agreement between predicted ultimate loads and the tests.



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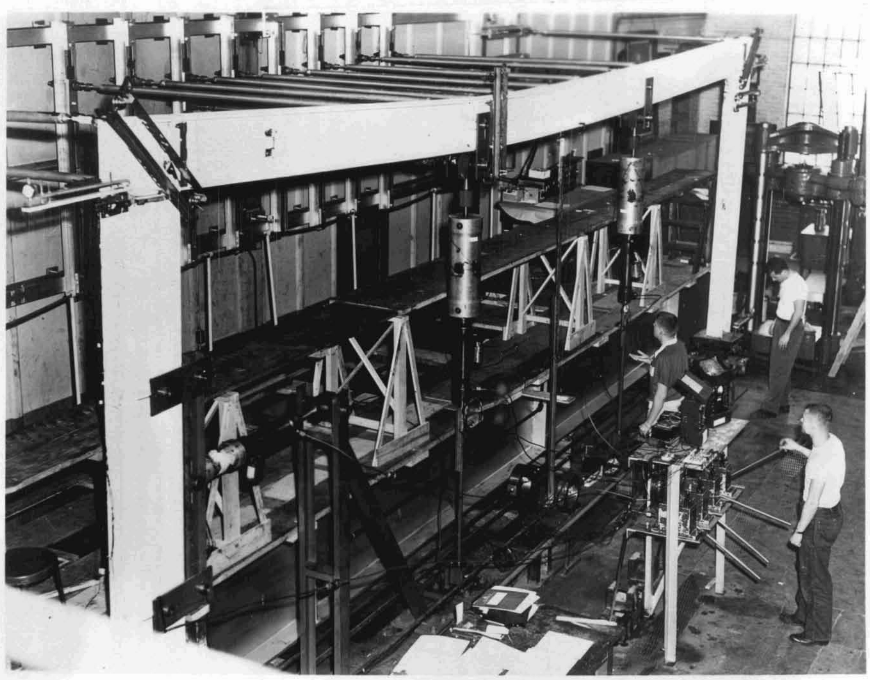


B

FIG.8. RIGID FRAME TESTS AT LEHIGH UNIVERSITY

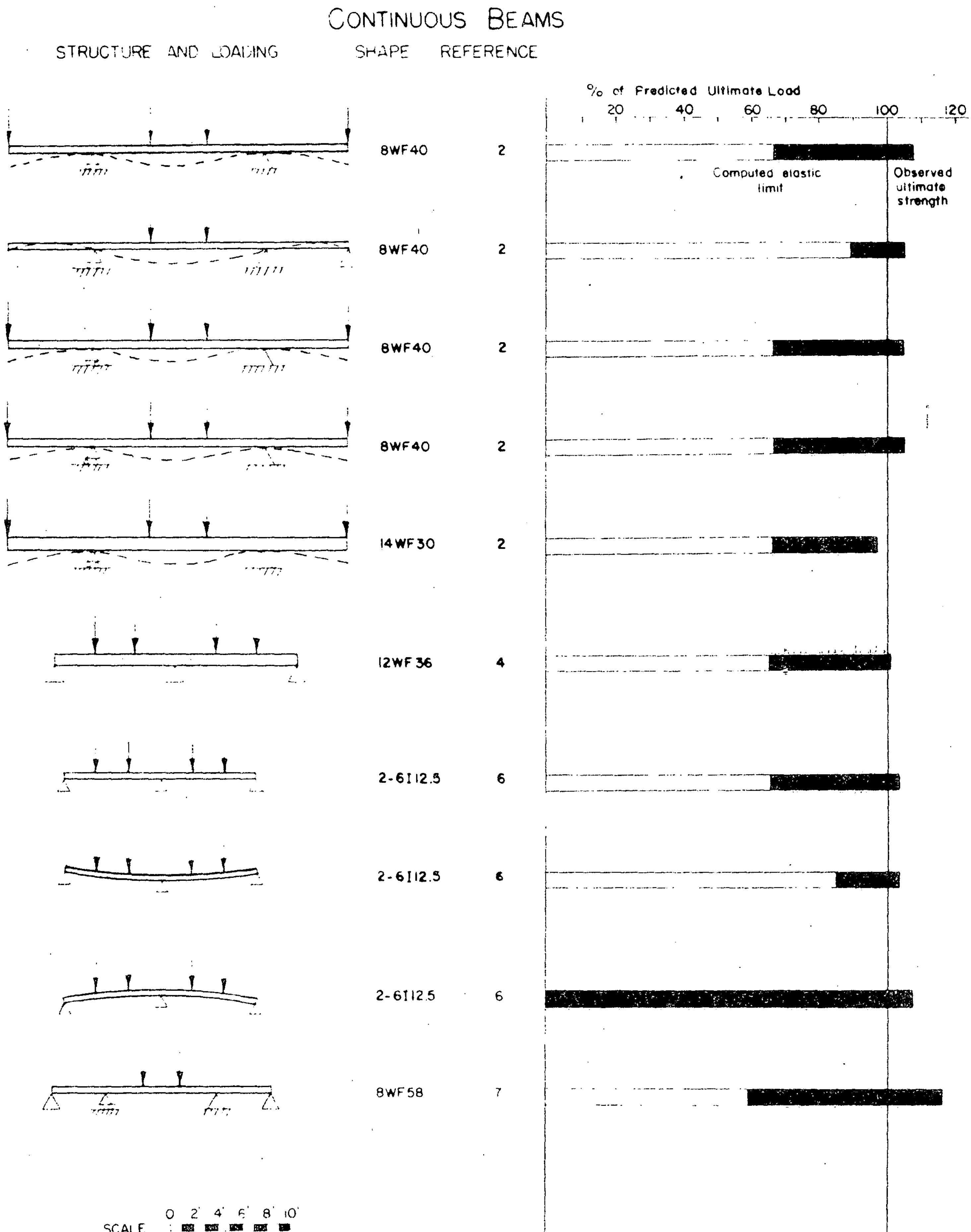


Fig. 9. Correlation between Test Results and Predictions by Plastic Design

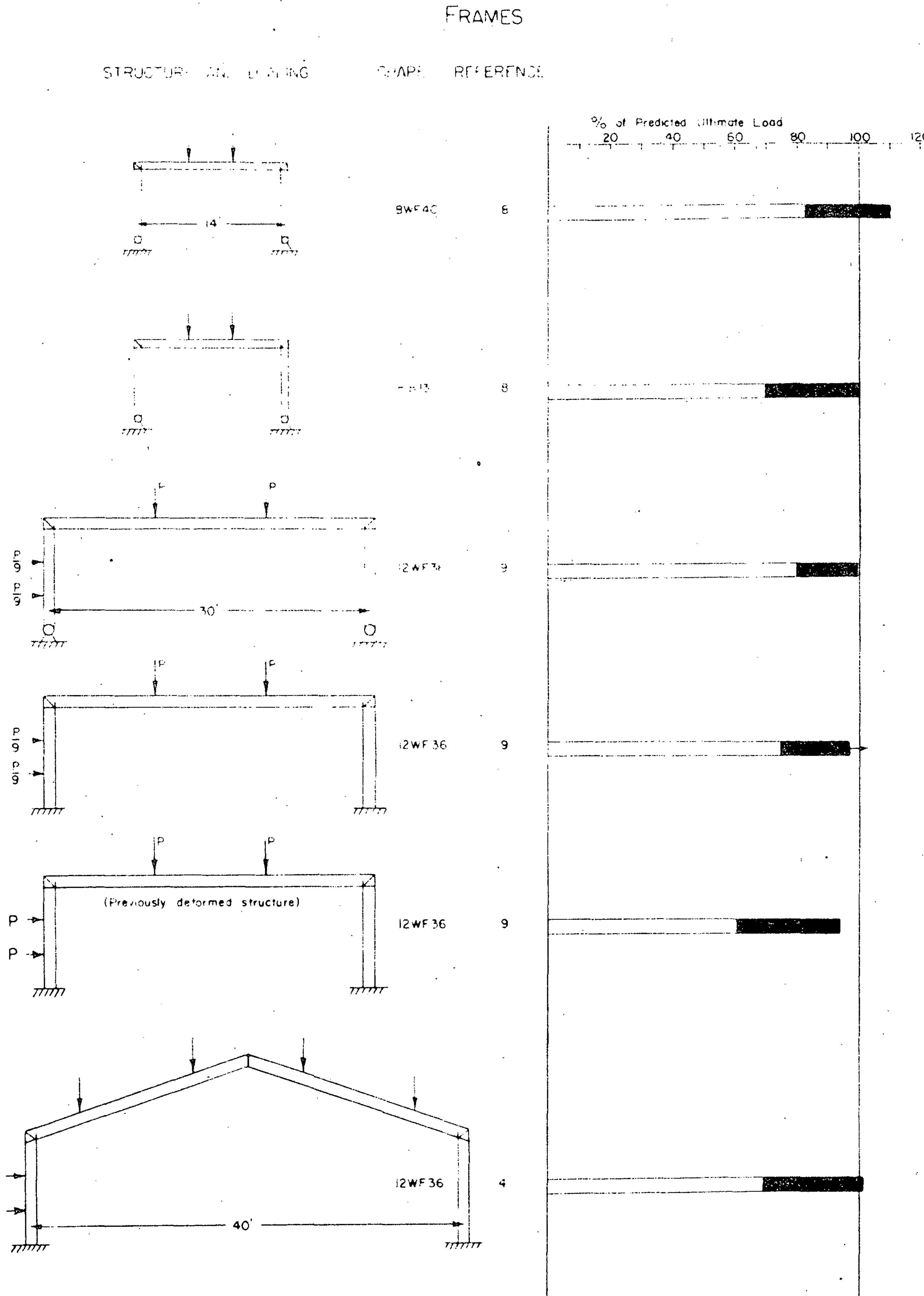


Fig. 10. Correlation between Test Results and Predictions by Plastic Design

APPLICATIONS OF PLASTIC DESIGN ~~AND RESTRICTIONS~~

As observed from the examples already shown, plastic design is especially applicable to continuous beams in buildings, including purlins, to one and two story rigid frame type industrial buildings, to tier type buildings, and generally to structures intended to absorb dynamic loads such as earthquakes, collision, and bomb blast. However, it is important to understand the restrictions or limitations of this new design method. These are described in the following paragraphs.

Materials

This design method is practically restricted to the use of that steel which is readily weldable, which will not fail in brittle fracture, and which exhibits a good range of plastic yielding after first obeying Hooke's law to a suitably high elastic limit. The steel must be resistant to the initiation and propagation of cracks under all service conditions. This is referred to as "notch toughness."

Notch Tougness

Notch toughness is basically a quality of the material selected, but it is dependent on temperature and it can be changed by bad fabrication procedures. Thus the material may loose its toughness and become brittle. Reference is made to the Three Rivers Bridge failure in Canada for such an example. Fabrication procedures may raise the temperature (called the "transition temperature") at which steel becomes brittle. It has been shown that the presence of internal stresses can raise the transition temperature. For example, in butt-welding together two beams of fully killed steel, the tests showed a transition temperature of -29°F in the "as-welded" condition while on companion specimens, one of which was stress-relieved and the other welded with preheat, the transition temperatures averaged -51°F.

It is well known that cold-working a steel causes a loss in notch toughness and raises the transition temperature. Tests, where the cold-working was done scientifically by inducing a fixed amount of pre-strain, (2%, 5% and 10%) showed an appreciable rise in the transition temperature, values of 60° rise for a 5% pre-strain being common.

Shearing and Punching

Shearing and punching are especially bad both for the cold working at the edges, and the possibility of causing small cracks at the sheared or punched edges which may initiate tearing.

Strain Aging

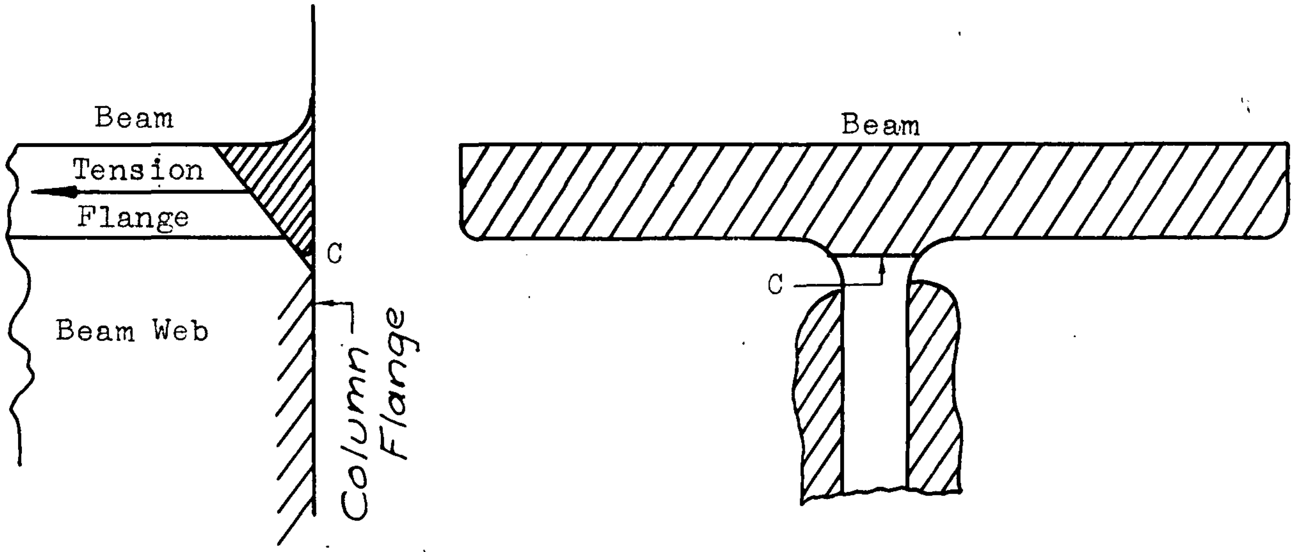
This is defined as a gradual change in properties occurring as a result of previous cold work. The time rate of change may be slow at room temperature, but may be accelerated by increasing the storage temperature. In fact, a temperature of 400 to 850°F can cause an immediate strain aging with a possible raising of the transition temperature to room temperature.

From the above statements concerning cold working and the accelerating effect of 400 - 850° F it may be concluded that alertness is required to prevent a change in the properties of the steel from one that is tough and ductible to one that may become brittle. Shearing or punching followed by the nearby application of the cutting torch or the electric arc can well cause embrittlement. On the favorable side is the fact that the effect of shearing can be overcome by machining off 1/16" of metal. When welding on sheared edges, the heat of welding raises ~~takes~~ the cold-worked steel above 1600°F and the entire effect of the cold working is removed. The problems, then, arise from sheared or punched edges which are left exposed and which are subject to sizeable stresses.

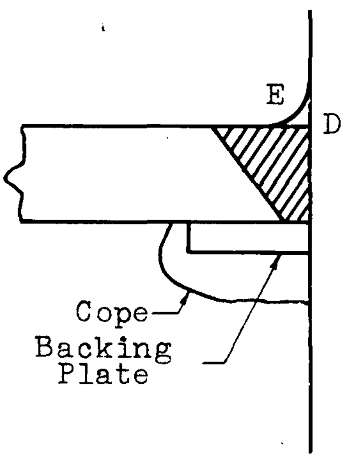
Stress Concentrations

Plastic design can hardly apply if, at a point where a plastic hinge is expected to form in an emergency, there is a stress concentration owing to faulty design. This phenomena is well known - and only one example is included. In Fig. , failure to cope and use a backing plate as in (A) may result in a stress concentration and possible crack starter at point C; while in Sketch B failure to add a small fillet at point E may cause a doubling of the stresses at D owing to the stress concentration factor.

From a study of the above limitations to the use of plastic design the following comments are offered:



A. Possible Crack Starter at "C" (No Backing Plate)



B. Cope and Use Backing Plate
Include Small Fillet at "E"

Figure 11 - EXAMPLES OF STRESS CONCENTRATIONS

1. If holes are to be provided drill them or else sub-punch and ream to remove the cold-worked material.

2. A sheared edge, if subjected to tension, could easily be the cause of a premature fracture.

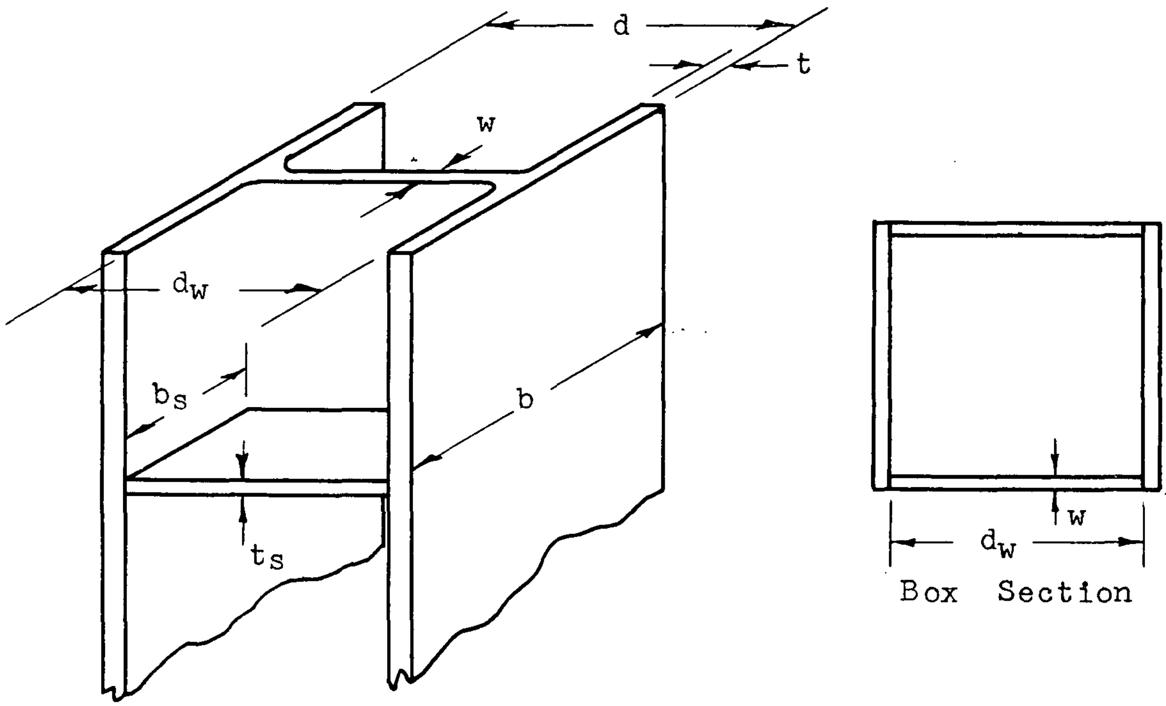
3. If welding is along the entire length of a sheared edge, theoretically there is no problem.

4. Do not peen welds - there are too many problems connected with it. Should peening be necessary, at least do not peen the last pass of welding.

5. If the application of heat is used to straighten a structural member, do not water-quench above the martensite temperature (about 600° F).

Local Instability

Plastic design requires that when the plastic hinge is forming there be no premature buckling of any of the structural parts. Research at Lehigh University has led to the maximum permissible width-thickness ratios shown on Fig. . Keeping the ratios within these limits is insurance that if a plastic hinge were to form because of overload, the hinge would maintain its full M_p through the required rotation angle as successive hinges form to the final ~~collapse~~ ultimate load.



Compression Elements
At Plastic Hinges

Flange of Rolled Beams
Flange Plates of Built-up Shapes $\Big\}$ $\frac{b}{t} \leq 17$

Stiffeners - - - - - - - - - - - - - $\frac{b_s}{t_s} \leq 8.5$

Column Webs if $P > 0.3\ P_y$
Flange Plates in Box Sections $\Big\}$ $\frac{d_w}{w} \leq 43$

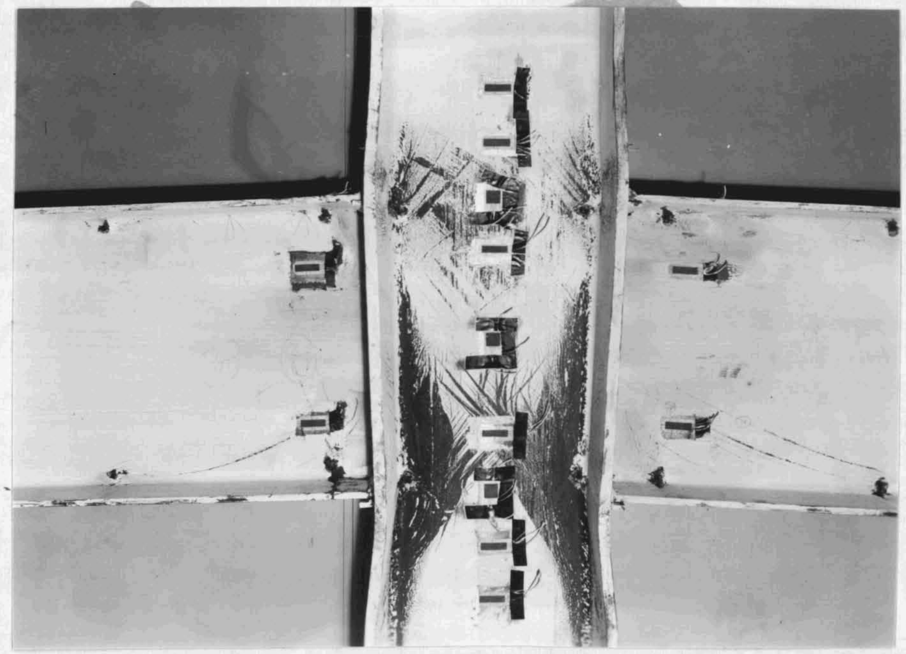
Figure 12 - PREVENTION OF LOCAL INSTABILITY

TIER BUILDINGS

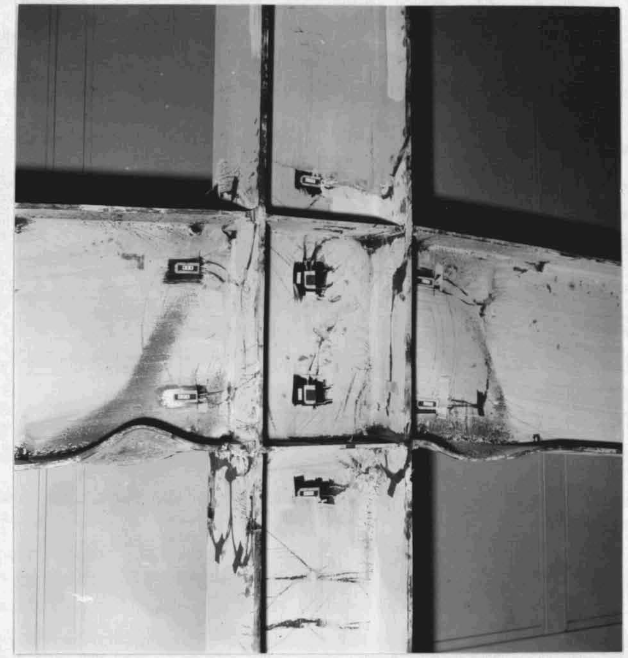
The principal problem involving plastic design of tier buildings is in the beam-to-column connections. Many tier buildings have such proportions or are provided with bracing such that wind moments do not enter into the design of these beam-column connections. This condition will be treated first.

The decision must be made, in designing a tier building, whether to cut the beams to length with the usual shop tolerances found in riveted practice and employ a seat and top plate type of connection or to cut the beams to exact length and weld directly to the columns. Direct welding, shown in Fig. 13, has the merit that the welds transmit the stress directly from beam to column, while in the seat-top plate connection the stress is transmitted twice by way of the connecting elements. Since the direct welding employs butt-welding at 20 Ksi and the welding on the seat and top plate involves at least half fillet welding at 13.6 Ksi the argument can be advanced that direct connections cut the required welding by over 50%. Later it will be shown that this is not entirely the case if one takes advantage of special techniques in designing the top plate connections.

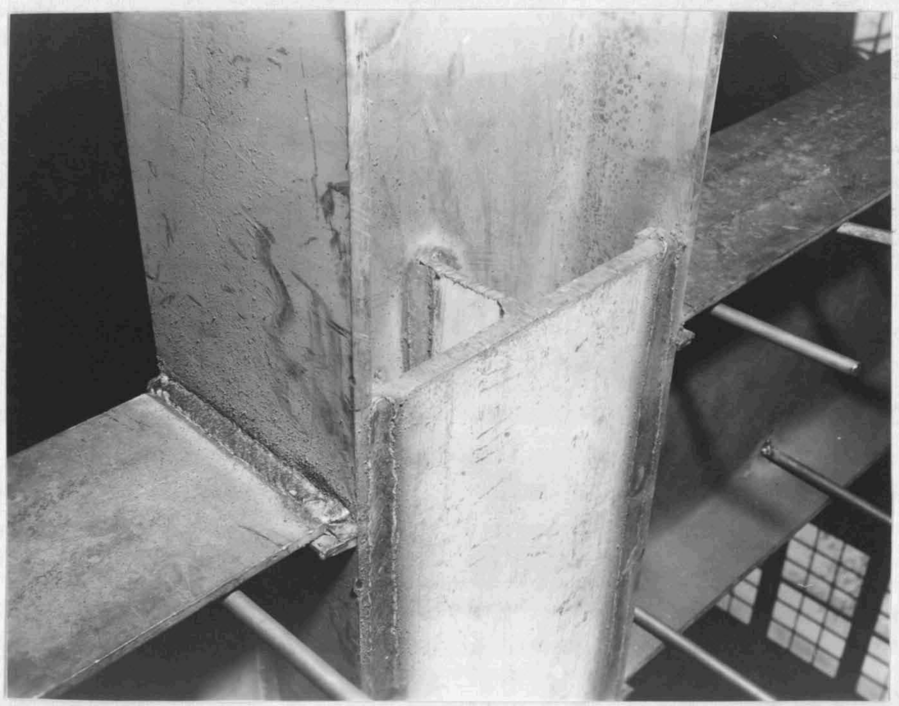
The four photographs in Fig. 13 show various means of effecting the connections. Since the objective of the tests was to obtain fundamental information on behavior, seat angles and other erection devices were omitted. Note in (A) that the test was made without column stiffeners. Since the full M_p of the beams was not realized



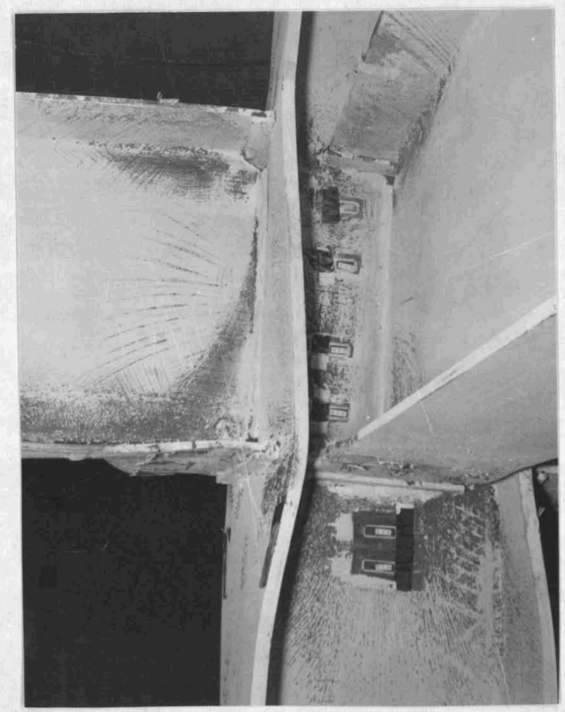
A



B



C



D

FIG. 13. BEAM-COLUMN CONNECTIONS.

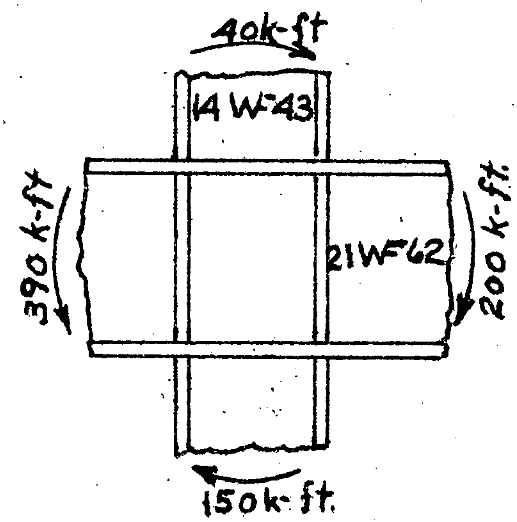
it may be inferred that the column web was deficient. In (B) and (C) stiffening is provided in two different ways; the horizontal plate stiffeners in (B) appear to be an efficient way to accomplish the mission of stiffening but certain problems appear should this be applied to a four-way connection. The vertical stiffening in (C) appears to be needlessly heavy but it does have the merit that it is all shop welding and much of the welding could be done by semi-automatic process. It is evident that (C) is excellent from the viewpoint of field erection of four-way connections. The four-way connection (D) would undoubtedly pose some field erection problems; erection seats, plates, or clips are evidently indicated.

THE AISC web-crippling formula that: $\frac{R}{w(t+2k)}$ must not exceed 24 ksi ~~must not exceed 24 ksi~~ is too conservative. As a temporary measure the 2 k in the formula can be changed to 5k; however, a report by the AISC presently in progress will probably propose a new formula. Should a column web be found deficient by the above formula (but modified as noted), the stiffeners must be designed to make up the deficiency. It is assumed that, if a stiffener is added to prevent web crippling, a similar stiffener will be placed in the column web at the beam tension flange.

The research to date indicates that the non-uniform distribution of stress in the butt welds connecting beam tension flange to column is not serious if the web crippling is not serious; therefore, until the above research is reported, the suggestion is made to use web crippling as the only criterion for determining the need for stiffeners but to require stiffeners in all doubtfull cases.

The need for web reinforcement at any connection can be determined by the formula:

$w = \dfrac{0.60\ M}{A}$ where "$\underline{w}$" is the required web thickness
"$\underline{M}$" is the algebraic sum of clockwise and counter-clockwise moments applied by members framing to opposite sides of the connection web boundary in k-ft.
"$\underline{A}$" is the area of connection web (out-to-out of flanges or stiffeners).



EXAMPLE: Is web reinforcement needed at the interior connection shown ?

Algebraic sum of moments on opposite sides of joint:

$390 - 200 = 190$ or $150 + 40 = 190$ k-ft.

For 14 WF 43, $w = 0.308''$
For 21 WF 62, $w = 0.40''$

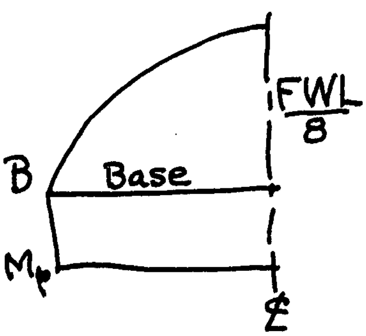
Run 21 WF through joint without reinforcement.

The design of the beams for direct welding, with no wind moment to be considered, is accomplished by formulas (1) and (9). Formula (9) would apply to a beam framing between an interior and an exterior column if it were desired to provide a flexible, or shear, connection to the exterior column.

Formula (1), $M_p = Fw\,L^2/16$, seems so daring if one has previously used the simple beam formula, $w\,L^2/8$, that a computation of deflection is included herewith for the 15 ft span beam with a 50 Kip load for which a 12WF27 was found satisfactory.

Procedure: 1. Establish the first and last hinges to form. The elastic curve will be smooth and continuous except at the first hinges to form, where there will be additional hinge angles.

2. Draw the elastic curve. Use area-moment, or similar method.



3. Draw tangent to elastic curve at midspan; project to left or right support and take moments about A or B of the intervening areas.

$$\text{Deflection} = \cancel{\frac{FWL^3}{384EI}}\ \frac{L^2}{EI}\left[\frac{5}{48}\frac{FWL}{8} - \frac{M_p}{8}\right]$$

$$\cancel{\frac{1.85 \times 50 \times 180^3}{384 \times 30{,}000 \times 183.4} = 0.22''}\ \frac{180^2}{30000 \times 204}\left[\frac{5}{48} \times \frac{185 \times 50 \times 180}{8} - \frac{1252}{8}\right]$$

$$= 0.32''$$

The computed deflection for the corresponding simple beam, a 16WF36, is ~~0.28"~~

$$\Delta = \frac{5}{384}\frac{FWL^3}{EI} = \frac{5 \times 1.85 \times 50 \times 180^3}{384 \times 30000 \times 446.3} = 0.53''$$

For those who prefer the top plate and seated type of connection reference is made for design procedures to a paper: "Welded Top Plate Beam-Column Connection" by R. Ford Pray and the speaker[21]. However, since the present subject is "Plastic Design" a modification is presented here strictly conforming to plastic design methods:

Procedure: 1. Select beam sizes that are somewhat conservative. The purpose is to have a midspan M_p somewhat large so the required end M_p can be smaller and thus reduce the welding. Note that:

$$M_B + M_{℄} = F\omega L^2/8. \tag{18}$$

2. Determine M_B from the above equation where $M_{℄} = M_p$ of selected beam.

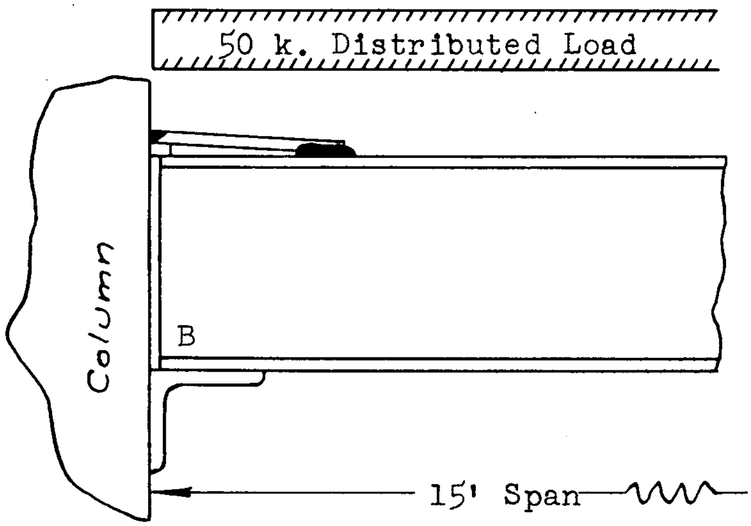
3. Tension in Top plate.

$$T = M_B/d \tag{19}$$

Area of top plate.

$$A = M_B/d\sigma y \tag{20}$$

For an example see Fig. 14



Given: Span and Load as shown

Design Top Plate

$$M_B + M_{℄} = \frac{FWL}{8}$$

$$= \frac{1.85 \times 50 \times 180}{8} = 2081$$
(kip-inches)

Use 14WF30 (M_p = 1554 k-in.)

$M_B = 2081 - 1554 = 527$ k-in.

Critical area of top plate

$$A = \frac{M_B}{d\sigma_y} = \frac{527}{14 \times 33} = 1.14 \text{ sq in.}$$

Make critical area 3.3" x 3/8 (A = 1.25 sq in.)

Welding Details to comply with AWS rule that welds are not to be overstressed.

Plate width at butt weld $= 3.3 \times \frac{33}{20} = 5.0"$

Plate tension at working load $= \frac{M_B}{d} = \frac{527}{14} = 37.7^k$

Use 5/16 F.W. at 3 k/in., Length = 37.7/3 = 12.6"

Choice of Top Plate (all plates 3/8" thick)

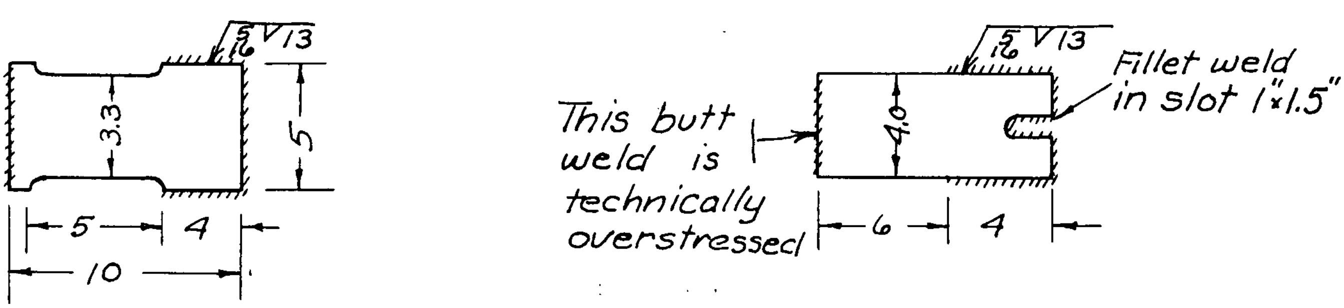


FIG. 14. TOP PLATE BEAM-COLUMN CONNECTION,- PLASTIC DESIGN

Tier Buildings - Effect of Wind

The application of plastic design to a combination of wind and gravity loads in a tier building rates much study and perhaps investigation, and, therefore, this subject will be inadequately covered here. The subject is scheduled for treatment by the team of researchers at Lehigh University who contributed so much to the acceptance of plastic design. As a temporary expedient the following design procedure is suggested and believed to be conservative:

1. Assuming no wind, determine M_p by methods already advanced. In this case F is taken as 1.85.

2. Find wind moments by any acceptable procedure.

3. Add wind moment to fixed end gravity moment ($wL^2/12$ for uniformly loaded beam). Multiply sum by an F factor of 1.4 to obtain required M_p.

4. Compare M_p's in (1) and (3). Use the larger moment to select the beam.

To prove that the above procedure is conservative - an example, taken from the Pray-Jensen paper (21), is reworked by plastic design using the above procedure. As noted in Fig. 15, a 15 ft. span beam is loaded with 50 Kips uniformly distributed and a wind moment at each end of 60 Kip-feet. Sketches (A) and (B) in the figure show the separate moment diagrams for the gravity load and the wind moments. Sketch (C), the composite moment diagram, shows a maximum working load moment of 122.5 Kip-ft. which gives a required M_p of

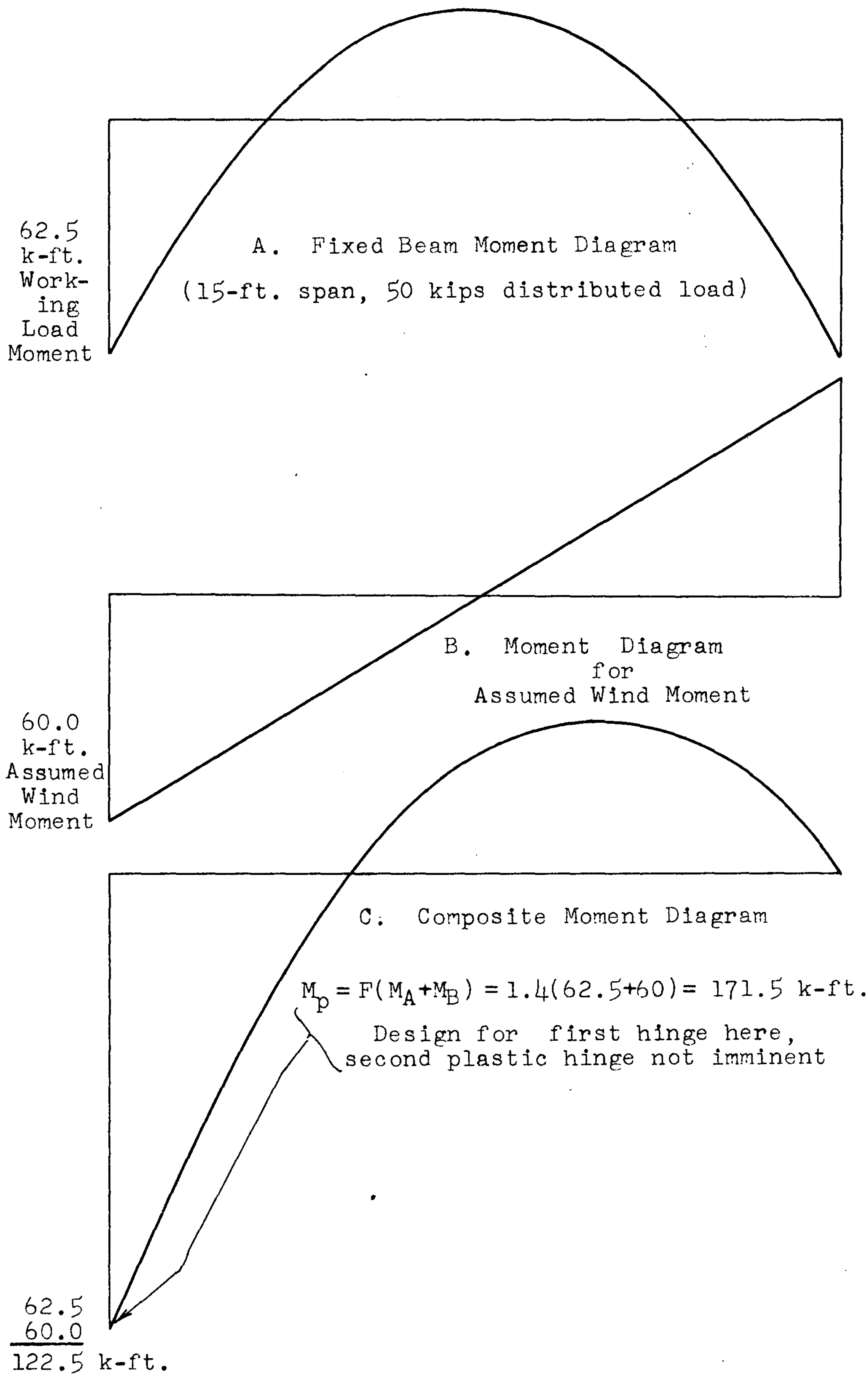


Figure 15 - Conservative Design For Gravity Plus Wind

1.4 x 122.5 = 171.5 k-ft. Use 16WF36. Note in this design that only the first plastic hinge is reached and that a considerable increase in either wind moment or gravity load is needed to bring about a true limit load condition, hence the statement that this is a conservative design.

In this paper an endeavor has been made to give a simple clear picture of "plastic design" and then to show how this new design method applies to structural welding. Welded joi~~ns~~t and plastic design are a natural pair, teaming up beautifully to provide strength, economy, and toughness. There are a few situations where high strength bolting may join forces with this team to effect further economies by encouraging shop welding of subassemblies and then field erecting by ~~bottling~~ bolting. In the case of beam-column connections the option was provided of directly welding the beams to the columns or of using the top plate and seat type of connection. In this latter case the field welding is kept to a minimum by selecting a beam slightly oversize by plastic design standards but very economical by elastic design standards and thereby requiring but a modest size of top plate for the end connection and but a modest amount of welding. The decision of which type of connection to use rests with the fabricator and depends much on his equipment. And lastly, in the case of tier buildings where both wind and gravity loads must be considered, a conservative design procedure - but still economical by elastic design standards, has been advanced as a stop gap until new studies yield more exact methods.

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Lastly, especial appreciation is expressed to Dr. Lynn Beedle for his constructive comments in preparation of this paper.

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